

Neutron Stars – II

Nuclear & Particle Physics of Compact Stars

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Collective Dynamics in High-Energy Collisions

Medium Properties, Chiral Symmetry and Astrophysical Phenomena

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Gerry Brown: “You can’t tell nature how to behave. Our job is to find out how, perhaps why.”

Evolution of a Nascent Neutron Star

$$\begin{aligned}
 \frac{dP}{dr} &= -G \frac{(m + 4\pi r^3 P)(\rho + P/c^2)}{r(r - 2Gm/c^2)} \\
 \frac{dm}{dr} &= 4\pi r^2 \rho; \quad \frac{d\mu}{dr} = 4\pi r^2 \rho_0 e^\Lambda \\
 \frac{d(N_\nu/\rho_0)}{d\tau} + e^{-\phi} \frac{\partial(L_n e^\phi)}{\partial\mu} &= S_n, \quad L_n = 4\pi r^2 F_n \\
 \frac{d(N_e/\rho_0)}{d\tau} &= -S_n \\
 \frac{d(E/\rho_0)}{d\tau} + P \frac{(1/\rho_0)}{d\tau} + e^{-2\phi} \frac{\partial(L_e e^{2\phi})}{\partial\mu} &= S_e; \quad L_e = 4\pi r^2 F_e
 \end{aligned}$$

- m := enclosed grav. mass ; • μ := enclosed rest mass
- ρ := mass-energy density ; • ρ_0 := baryon rest mass density
- N_ν := ν_e number density ; • L_n := lepton luminosity
- F_n := lepton flux ; • S_n := ν_e source term
- Metric: $ds^2 = -e^{2\phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\Omega^2$
- t := universal coordinate time; • τ := local proper time

$$e^\phi = \sqrt{-g_{00}} \quad e^{2\Lambda} = 1/(1 - 2Gm/rc^2) \quad e^{-\phi} d/dt = d/d\tau$$

Composition of Dense Stellar Matter

- Crustal Surface :

electrons, nuclei, dripped neutrons, ⋯ set in a lattice
new phases with lasagna, spaghetti, ⋯ like structures

- Liquid (Solid?) Core :

n, p, Δ, \dots leptons: $e^\pm, \mu^\pm, \nu'_e s, \nu'_\mu s$

$\Lambda, \Sigma, \Xi, \dots$

K^-, π^-, \dots condensates

u, d, s, \dots quarks

- Constraints :

1. $n_b = n_n + n_p + n_\Lambda + \dots$:

baryon # conservation

2. $n_p + n_{\Sigma^+} + \dots = n_e + n_\mu$:

charge neutrality

3. $\mu_i = b_i \mu_n - q_i \mu_\ell$:

energy conservation

\Rightarrow

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e \quad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e$$

\Rightarrow

$$\mu_{K^-} = \mu_e = \mu_\mu = \mu_n - \mu_p$$

\Rightarrow

$$\mu_d = \mu_u + \mu_e = \mu_s = (\mu_n + \mu_e)/3$$

$$\mu_u = (\mu_n - 2\mu_e)/3$$

NEUTRINO TRAPPED STARS

(Newly-Born neutron stars)

- Entropy/baryon $\sim 1 - 2$
- Leptons/baryons $Y_{L\ell} = Y_\ell + Y_{\nu\ell}$, ($\ell = e, \mu$ and τ)
conserved on dynamical timescales of collapse
- Neutrinos trapped !
- Chemical equilibrium

$$\Rightarrow \boxed{\mu_i = b_i \mu_n - q_i (\mu_\ell - \mu_{\nu\ell})}$$

b_i : baryon #

q_i : baryon charge

$\mu_{\nu\ell}$: neutrino chemical potential

- Collapse calculations

$$\Rightarrow Y_{Le} = Y_e + Y_{\nu_e} \simeq 0.4$$
$$Y_{L\mu} = Y_\mu + Y_{\nu_\mu} \simeq 0.0$$

Prakash et al., Phys. Rep. 280, 1 (1997).

Why Neutrinos Get Trapped

- Roughly, the scattering mean free path of ν 's in dilute matter is

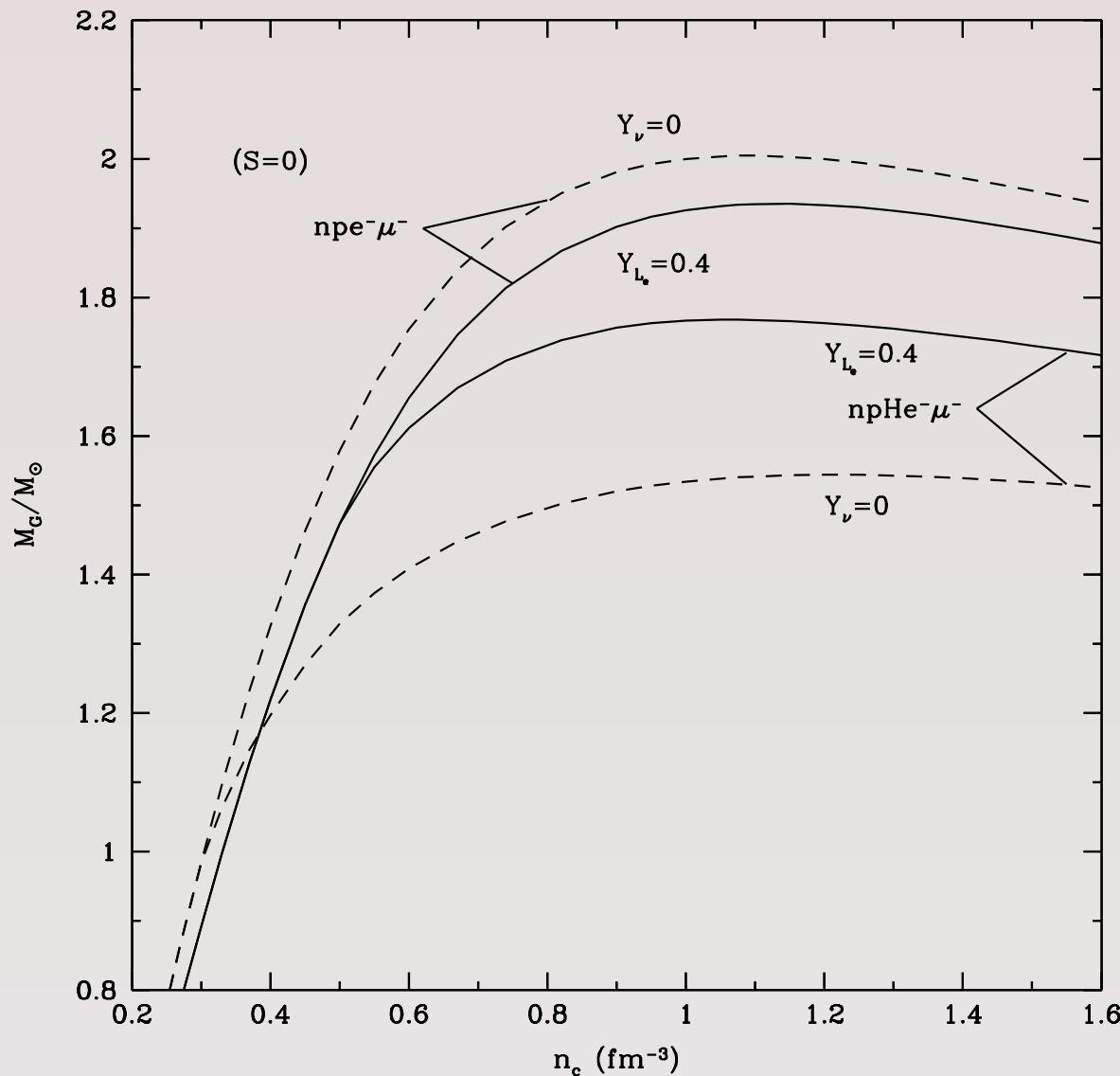
$$\begin{aligned}\lambda &= \left(n_n G_F^2 E_\nu^2\right)^{-1} \\ &\simeq 2 \times 10^5 \left(\frac{\text{MeV}}{E_\nu}\right)^2 \text{ cm} \quad \text{for } n_n = 0.16 \text{ fm}^{-3}\end{aligned}$$

For $E_\nu \sim 200 \text{ MeV}$,

Mean free path $\lambda \simeq 5 \text{ cm} \ll R \sim (10 - 100) \text{ km}$

- Effects of degeneracy (Pauli exclusion) and interactions increase λ by an order of magnitude and are important, but affect trapping only quantitatively.
- Even the elusive neutrinos are trapped in matter, albeit transiently, in the supernova environment!

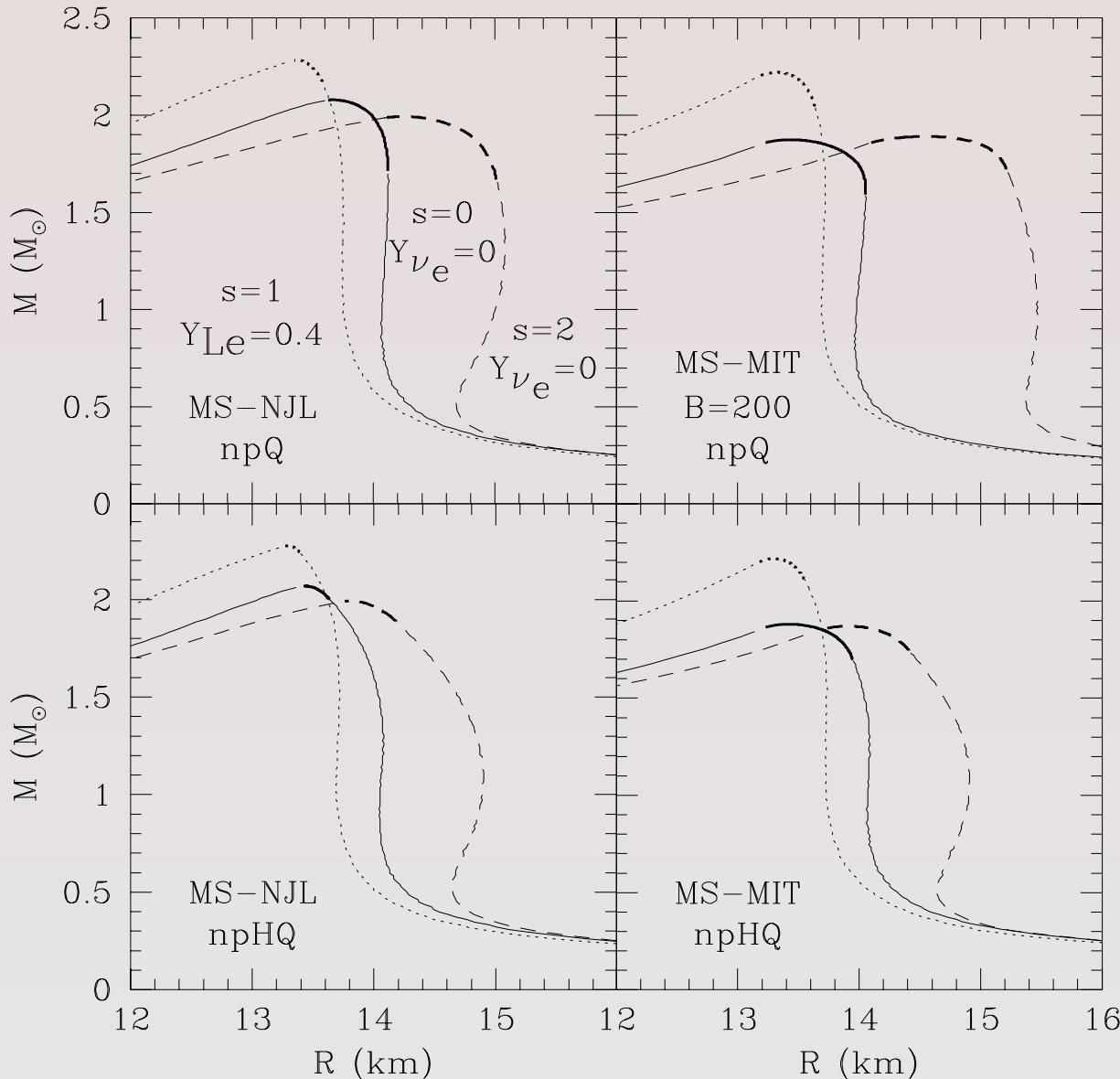
Neutrino Trapping in Hyperonic Matter



- ▶ $npe^- \mu^-$: Trapped ν 's decrease the maximum mass
- ▶ $npHe^- \mu^-$: Trapped ν 's increase the maximum mass
- ▶ ν -trapping delays the appearance of H 's till larger densities are reached

Prakash et al., Phys. Rep. 280, 1 (1997).

Neutrino Trapping in Quark Matter

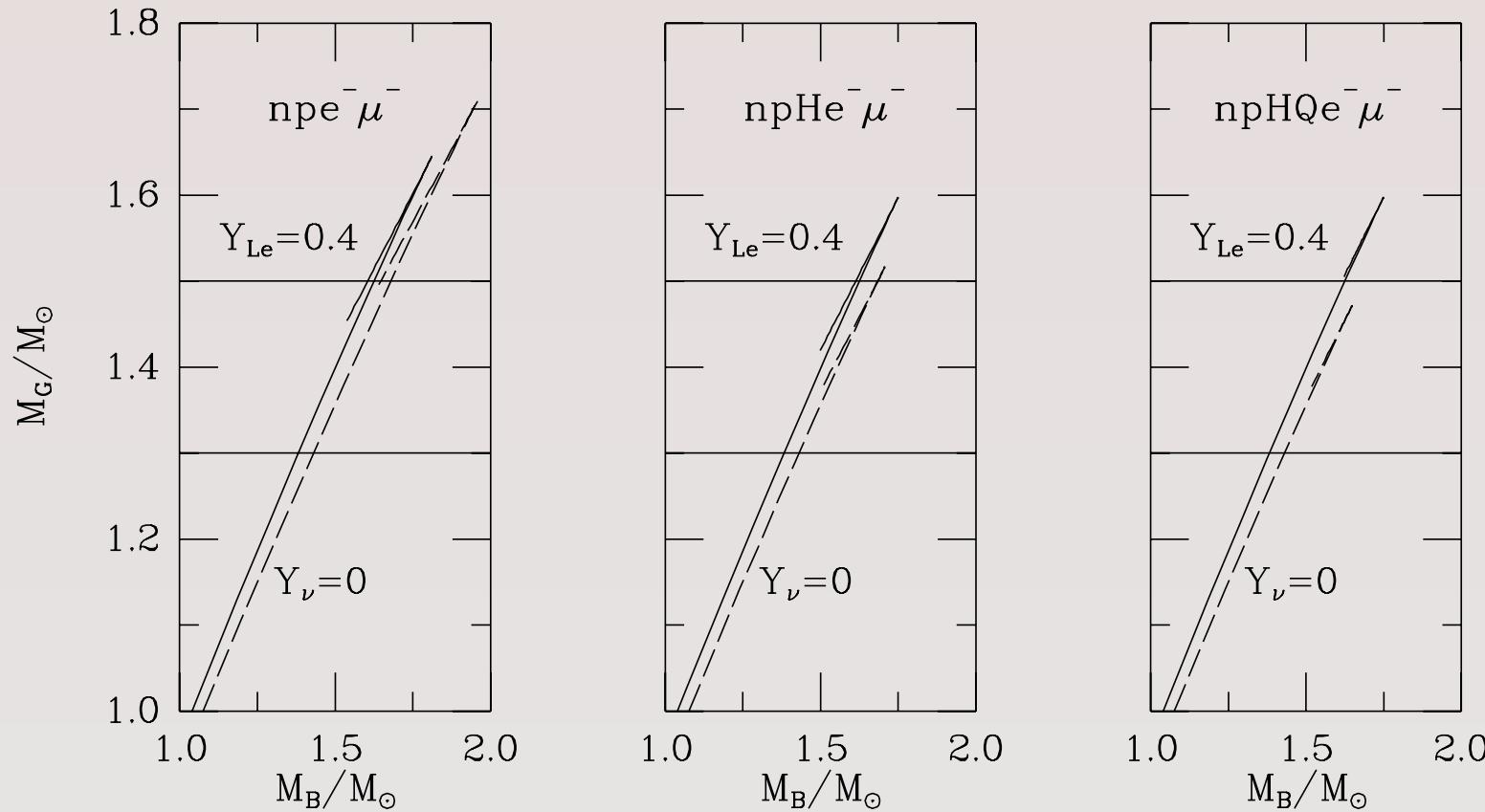


- Dark portions show mixed phase regions
- ν -free stars with $s = 2$ have smaller maximum masses than those with $s = 0$
- With ν trapping a range of masses are metastable

Steiner, Prakash & Lattimer, Phys. Lett. B486, 239 (2000).

Neutrino Trapping & Metastability

If the initial protoneutron star maximum mass is greater than that of the final configuration, collapse to a black hole is inevitable.



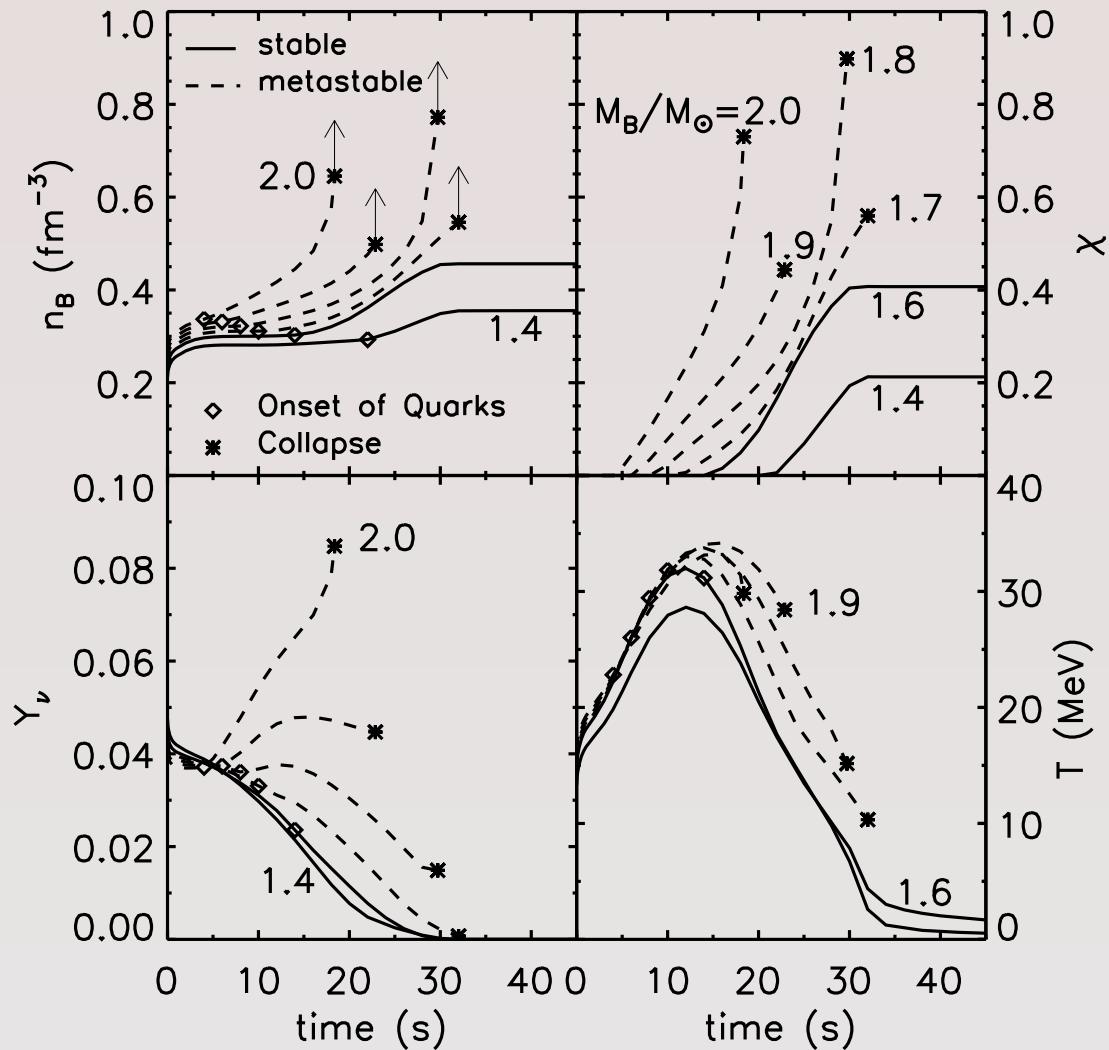
Ellis, Lattimer & Prakash, Comm. Nucl. & Part. Phys. 22, 63 (1996).

Metastability

1. Appearance of hyperons, bosons and/or quarks is delayed to higher density in neutrino-trapped (lepton-rich) matter.
2. Nascent neutron stars, with negatively charged strongly interacting particles, have larger maximum masses than their cold catalyzed counterparts; a reversal in behavior from matter containing only neutrons, protons and leptons.
3. Above permits existence of metastable young stars that could collapse to black holes during deleptonization.
4. In all cases, effects of entropy (of order 1 or 2) on the maximum mass are small in comparison to effects of neutrino trapping.

Prakash et al., Phys. Rep. 280, 1 (1997).

Evolution of Matter with Quarks



- As ν 's leave, the central density & mass of nucleons-only stars stabilize
- Massive enough stars with quarks end up as black holes upon deleptonization

Pons, Steiner, Prakash & Lattimer, Phys. Rev. Lett. 86, 10 (2001).

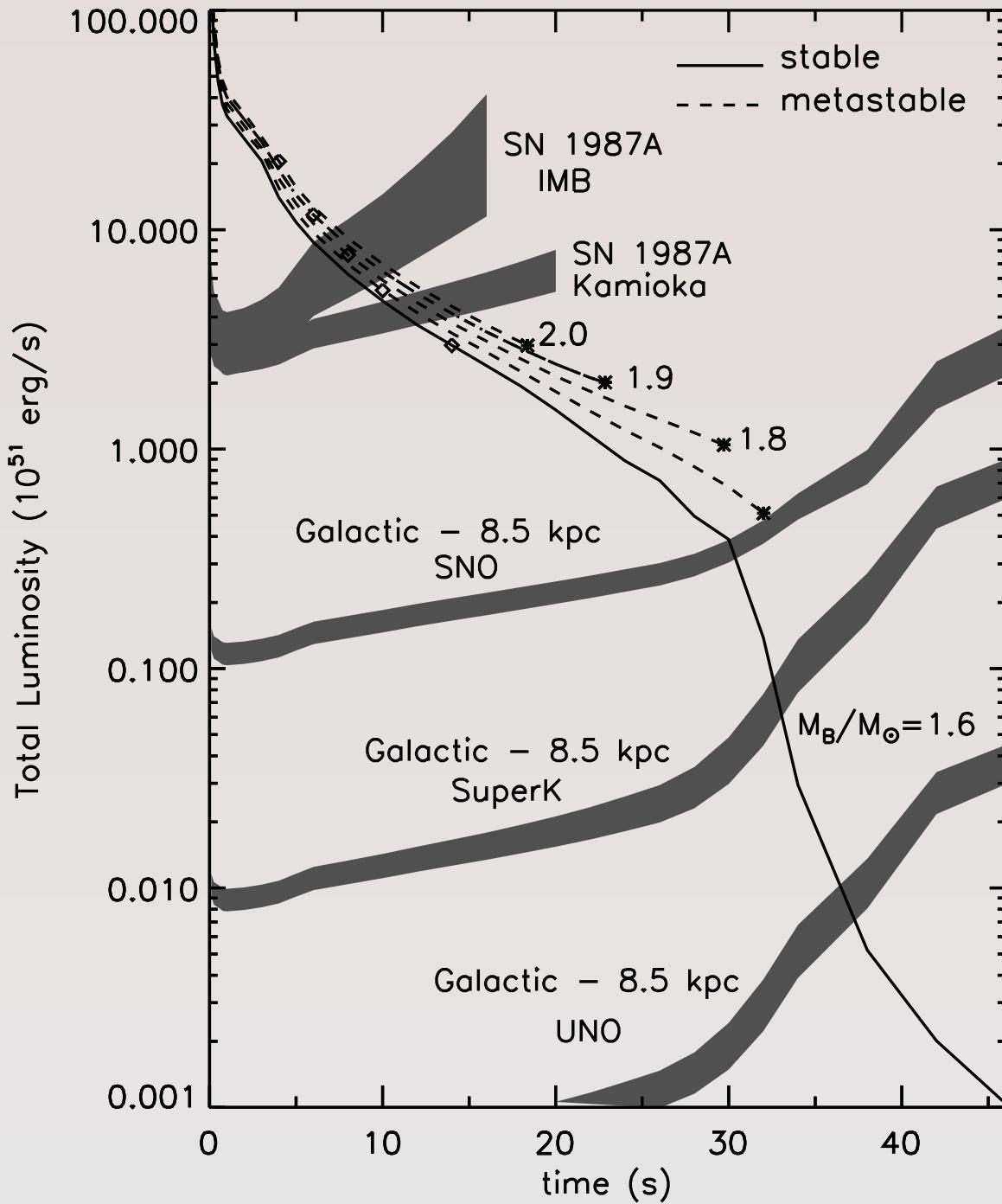
Signals in Terrestrial Detectors

(Bare-Bone Approach)

$$\frac{dN}{dt} = \left(\frac{R_\nu^\infty}{D} \right)^2 \frac{c\sigma_0 n_p}{8\pi(\hbar c)^3} \mathcal{M} \int_{E_{th}}^{\infty} E_\nu^4 f(E_\nu, T_\nu^\infty) W(E_\nu) dE_\nu$$

- ▶ $R_\nu^\infty = e^{-\phi_s} R_\nu$: Radius of the neutrinosphere
- ▶ D : Distance to supernova
- ▶ $\sigma_0 = 9.3 \times 10^{-44}$ cm² : $\bar{\nu}_e p$ cross section
- ▶ $n_p = 6.7 \times 10^{28}$: Free protons per kiloton of water
- ▶ \mathcal{M} : Detector mass in kilotons
- ▶ $T_\nu^\infty = e^{\phi_s} T_\nu$: Neutrino temperature at freezeout
- ▶ $e^{\phi_s} = \left(1 - \frac{2GM_G}{R_\nu c^2} \right)^{1/2}$: Redshift factor
- ▶ E_{th} & $W(E_\nu)$: Detector threshold & efficiency

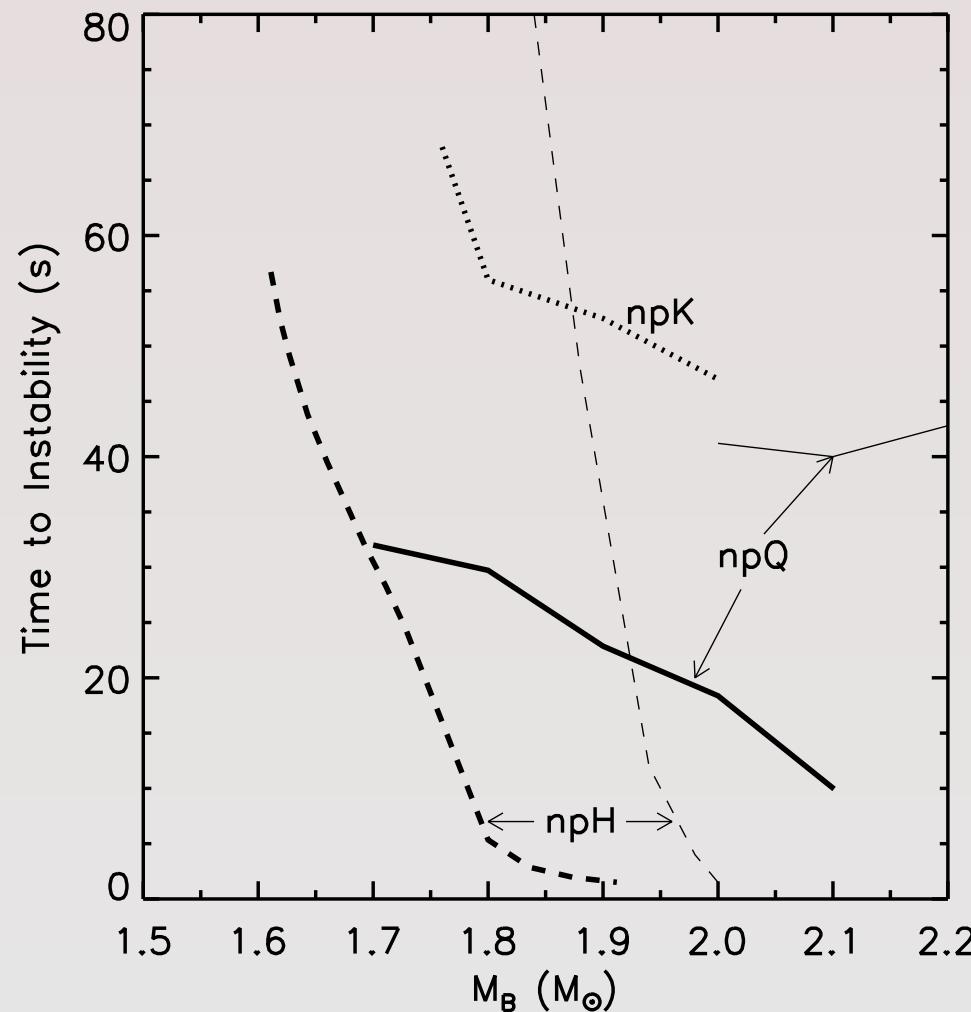
Neutrino Luminosities



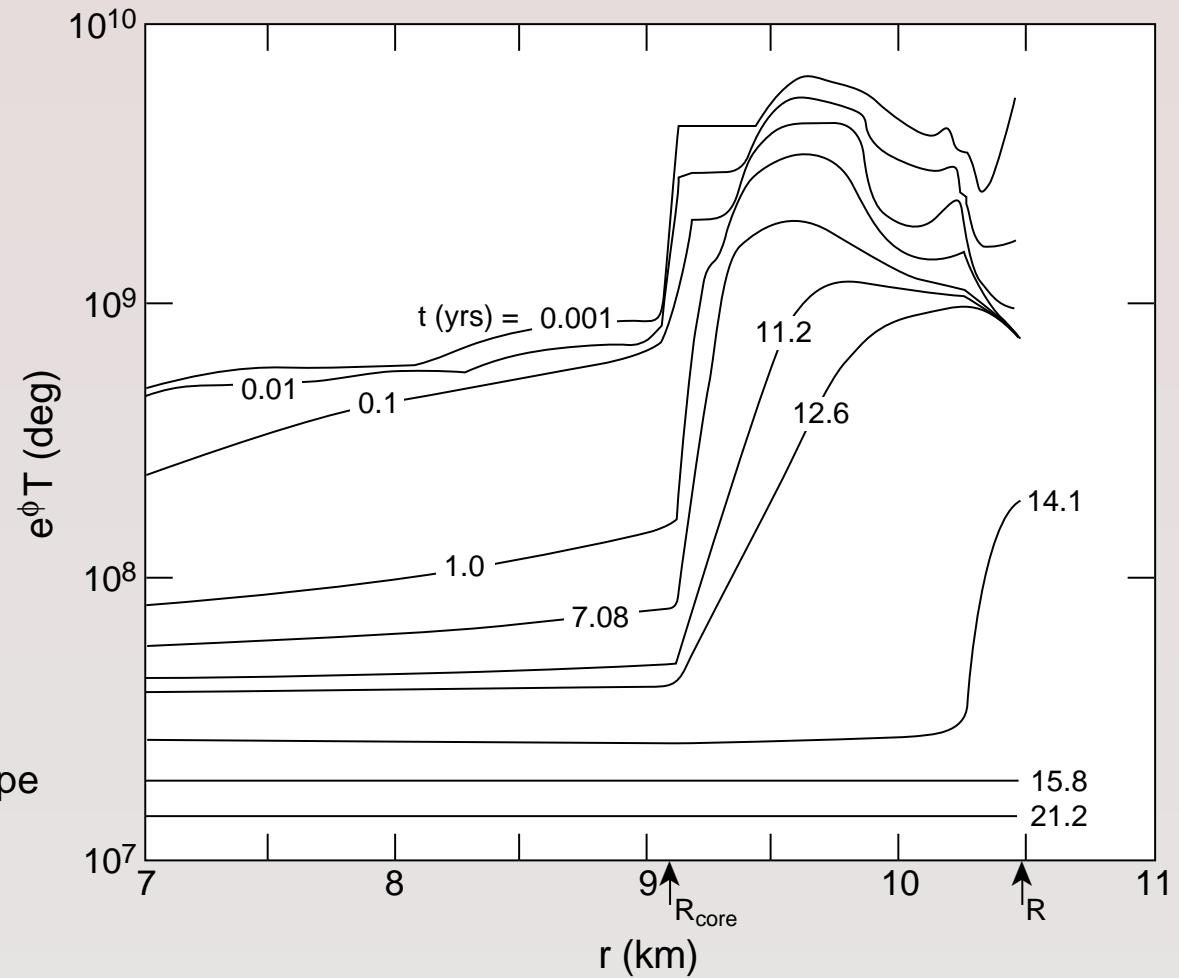
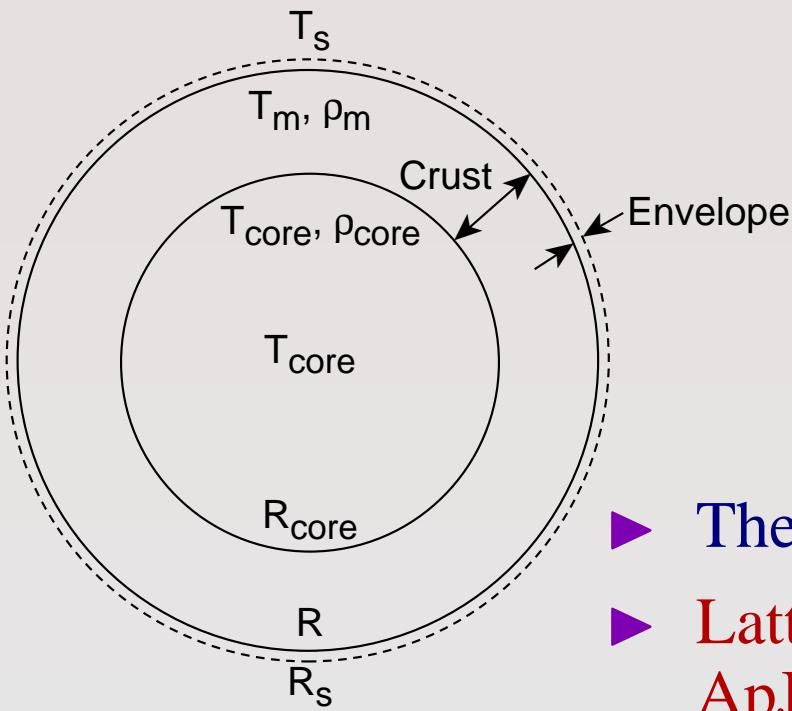
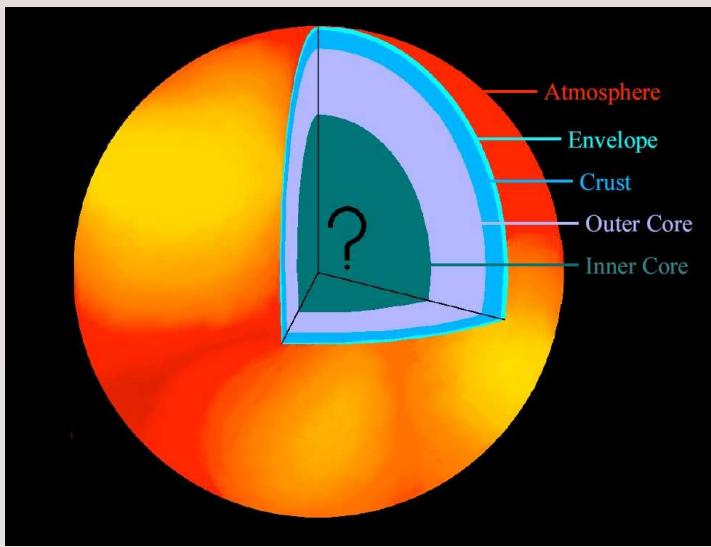
- ▶ Early detectors lacked sensitivity to test if SN 1987A ended up as a black hole.
- ▶ Current & future detectors can do better in the case of a future event.
- ▶ Prakash et al., Ann. Rev. Nucl. & Part. Sci. 51, 295 (2001).
- ▶ Future work: Luminosities in different ν -flavors.

Time to Instability

- ▶ Observation of metastability would signal the presence of exotica
- ▶ Frequency of galactic SN: 1 per (30-50) yr



Thermal Evolution



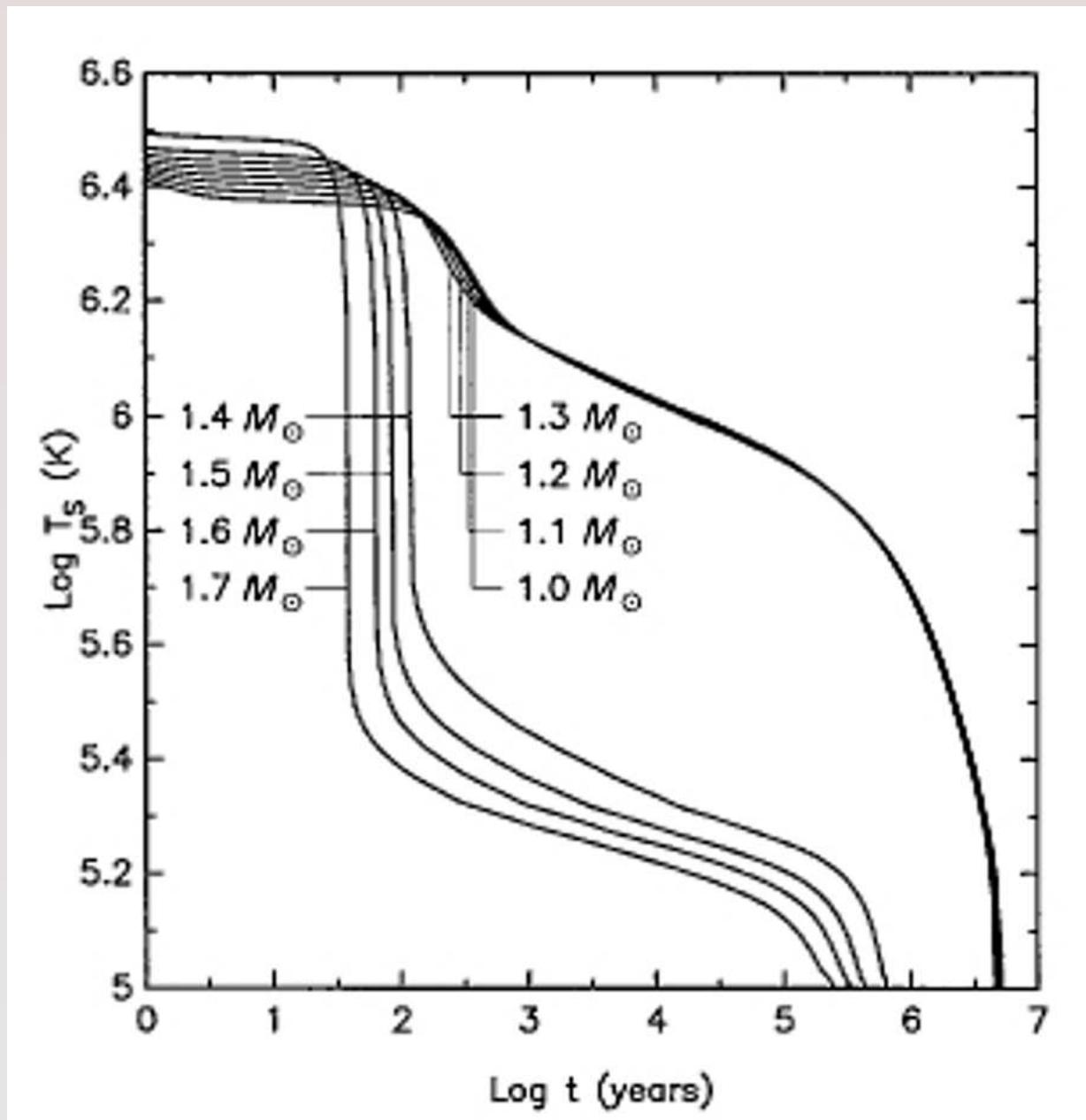
- The star becomes isothermal in tens of years.
- Lattimer, Van Riper, Prakash & Prakash, ApJ 425, 802 (1993).

Neutrino Emissivities

Name	Process	Emissivity (erg s ⁻¹ cm ⁻³)	References
Modified Urca	$n + n' \rightarrow n + p + e^- + \bar{\nu}_e$ $n' + p + e^- \rightarrow n' + n + \nu_e$	$\sim 10^{20} T_9^8$	Friman & Maxwell 1979
Kaon Condensate	$n + K^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + K^- + \nu_e$	$\sim 10^{24} T_9^6$	Brown et al., 1988
Pion Condensate	$n + \pi^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + \pi^- + \nu_e$	$\sim 10^{26} T_9^6$	Maxwell et al., 1977
Direct Urca	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\sim 10^{27} T_9^6$	Lattimer et al., 1991
Hyperon Urca	$B_1 \rightarrow B_2 + l + \bar{\nu}_l$ $B_2 + l \rightarrow B_1 + \nu_l$	$\sim 10^{26} T_9^6$	Prakash et al., 1992
Quark Urca	$d \rightarrow u + e^- + \bar{\nu}_e$ $u + e^- \rightarrow d + \nu_e$	$\sim 10^{26} \alpha_c T_9^6$	Iwamoto 1980

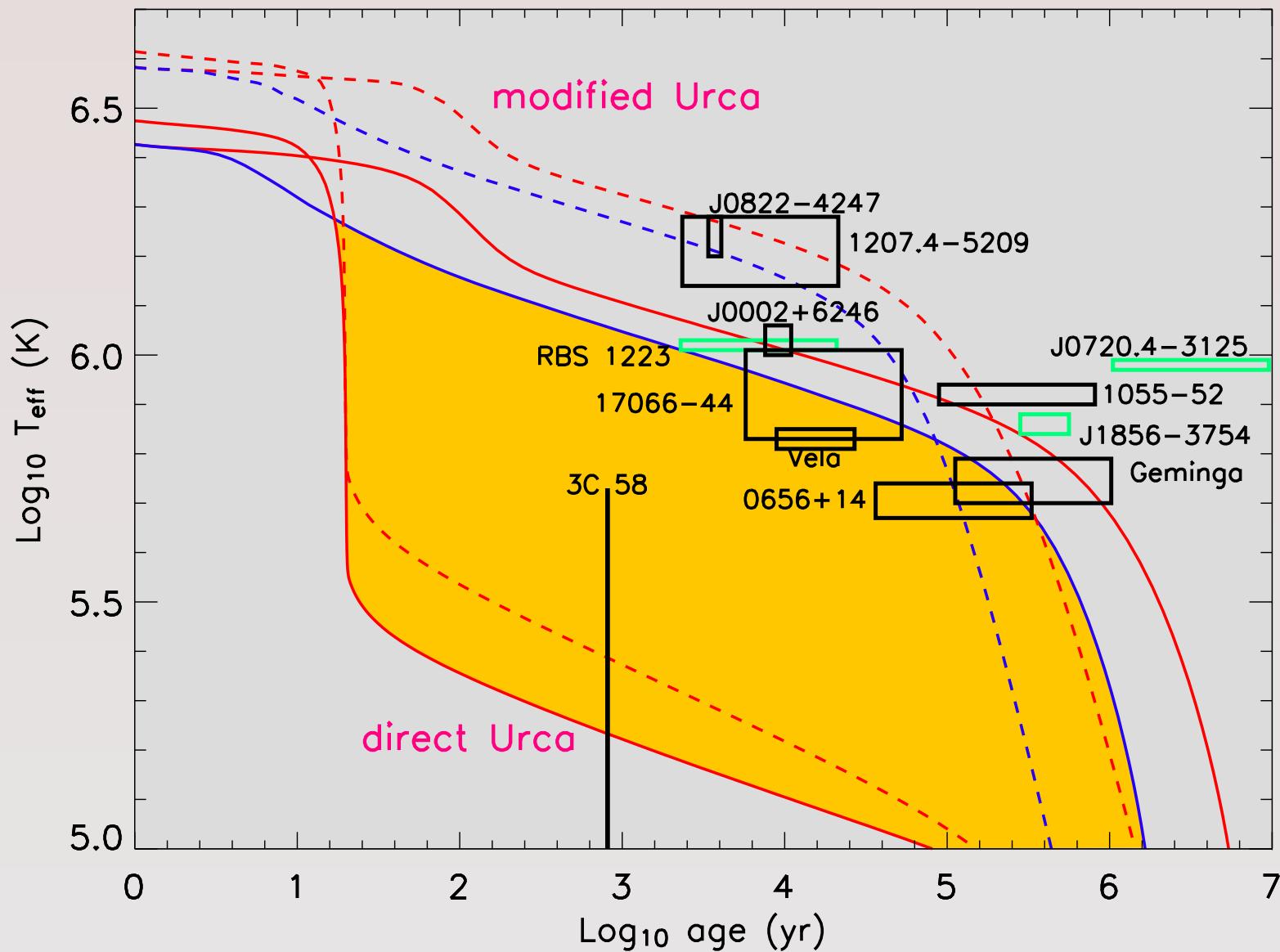
T_9 : Temperature in units of 10^9 K.

Direct versus Modified Urca



- ▶ Unlike MUrca, DURca exhibits threshold effects.
- ▶ Superfluidity abates DURca cooling.
- ▶ Page & Applegate, ApJ 394, L17 (1992).
- ▶ Cooper pair breaking & reformation affects both DURca & MUrca.

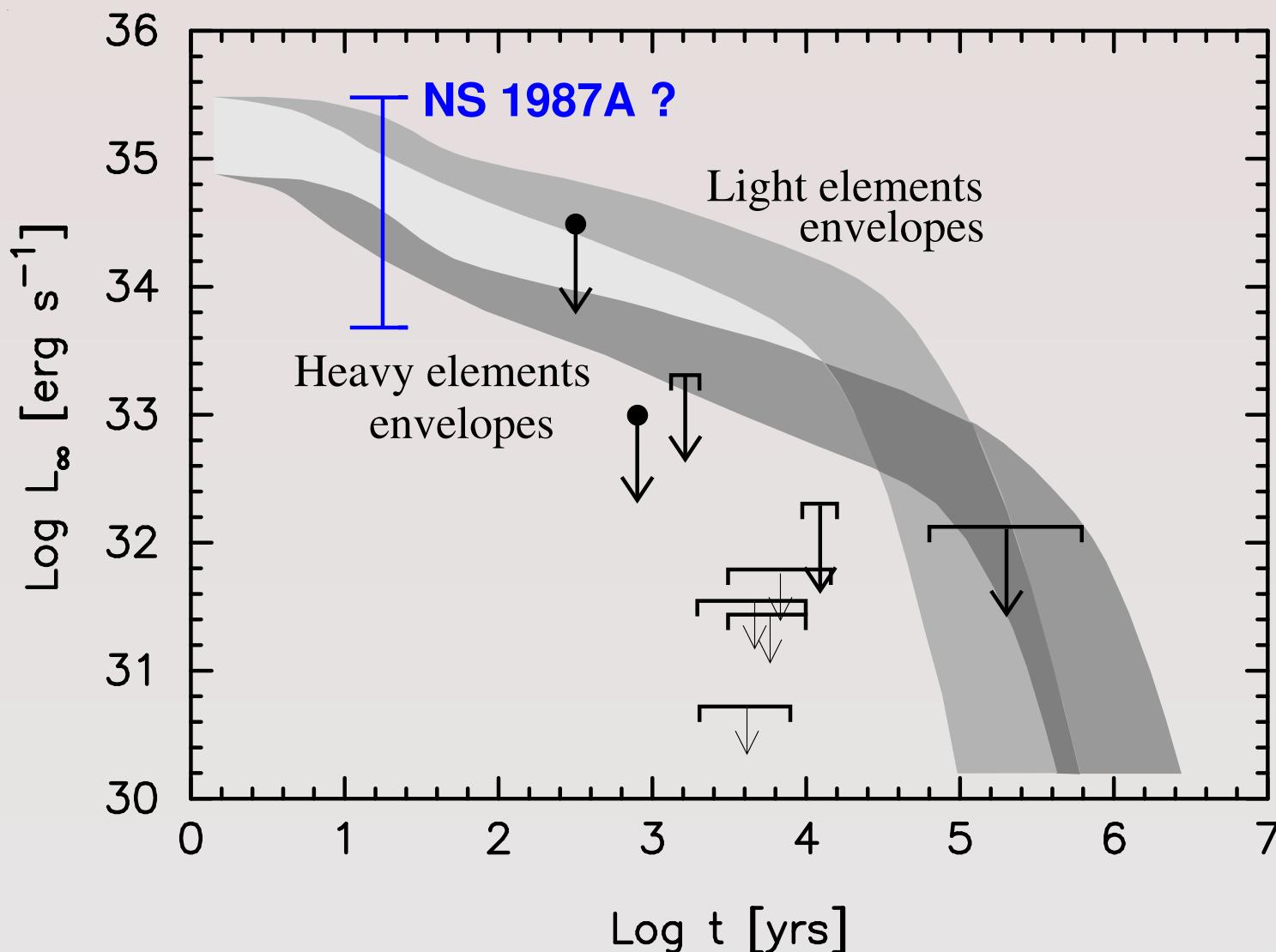
Inferred Surface Temperatures



Lattimer & Prakash , Science 304, 536 (2004).

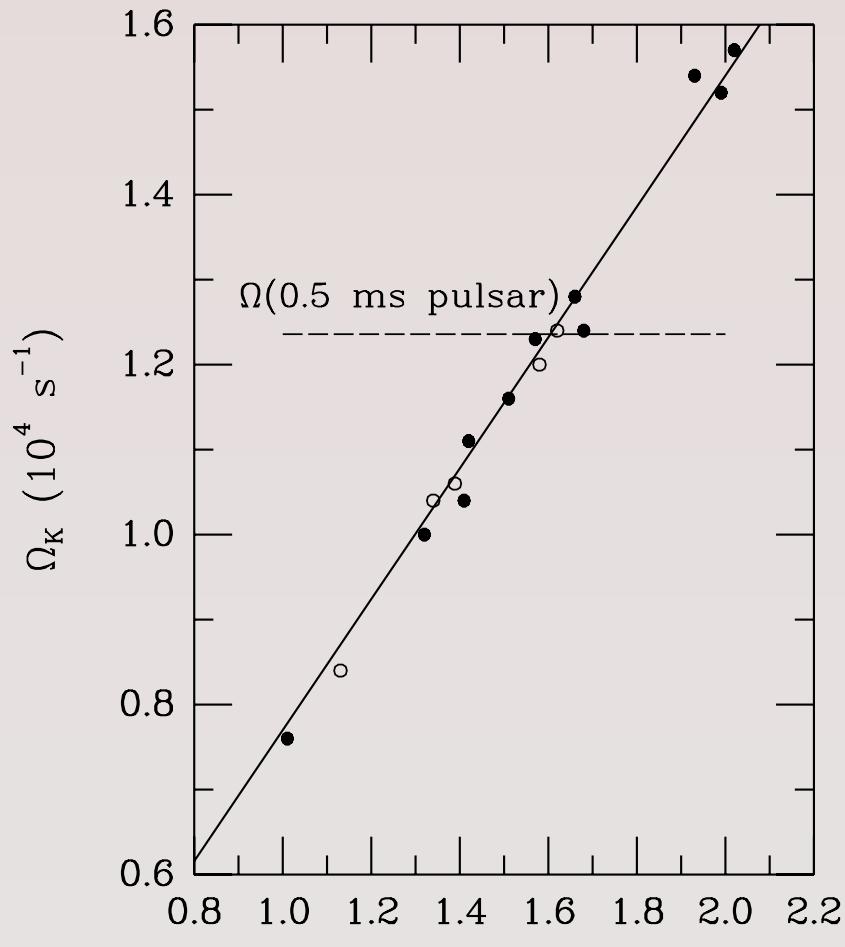
New Cold Objects

- Several cases fall below the “Minimal Cooling” paradigm & point to enhanced cooling, if these objects correspond to neutron stars.



Page, Lattimer, Prakash & Steiner, ApJS 155, 623 (2004).

How fast can rotation be?



General Relativity:

$$\Omega_K = 7.7 \times 10^3 \text{ s}^{-1} \times SF.$$

$$SF = \left(\frac{M_{max}}{M_\odot} \right)^{1/2} \left(\frac{R_{max}}{10 \text{ km}} \right)^{-3/2}$$

Maximum radius:

$$R_{max} = 10.3 \text{ km}$$

$$\times \left(\frac{1 \text{ kHz}}{\nu} \right)^{2/3} \left(\frac{M}{M_\odot} \right)^{1/3}$$

M: Star of given mass

M_{max} and R_{max} refer to the spherical configuration.

Lattimer & Prakash , Science 304, 536 (2004).

Moment of inertia (I) measurements

Spin precession periods:

$$P_{p,i} = \frac{2c^2 a P M (1 - e^2)}{G M_{-i} (4M_i + 3M_{-i})}.$$

Spin-orbit coupling causes a periodic departure from the expected time-of-arrival of pulses from pulsar A of amplitude

$$\delta t_A = \frac{M_B}{M} \frac{a}{c} \delta_i \cos i = \frac{a}{c} \frac{I_A}{M_A a^2} \frac{P}{P_A} \sin \theta_A \cos i$$

P : Orbital period a : Orbital separation e : Eccentricity

$M = M_1 + M_2$: Total mass

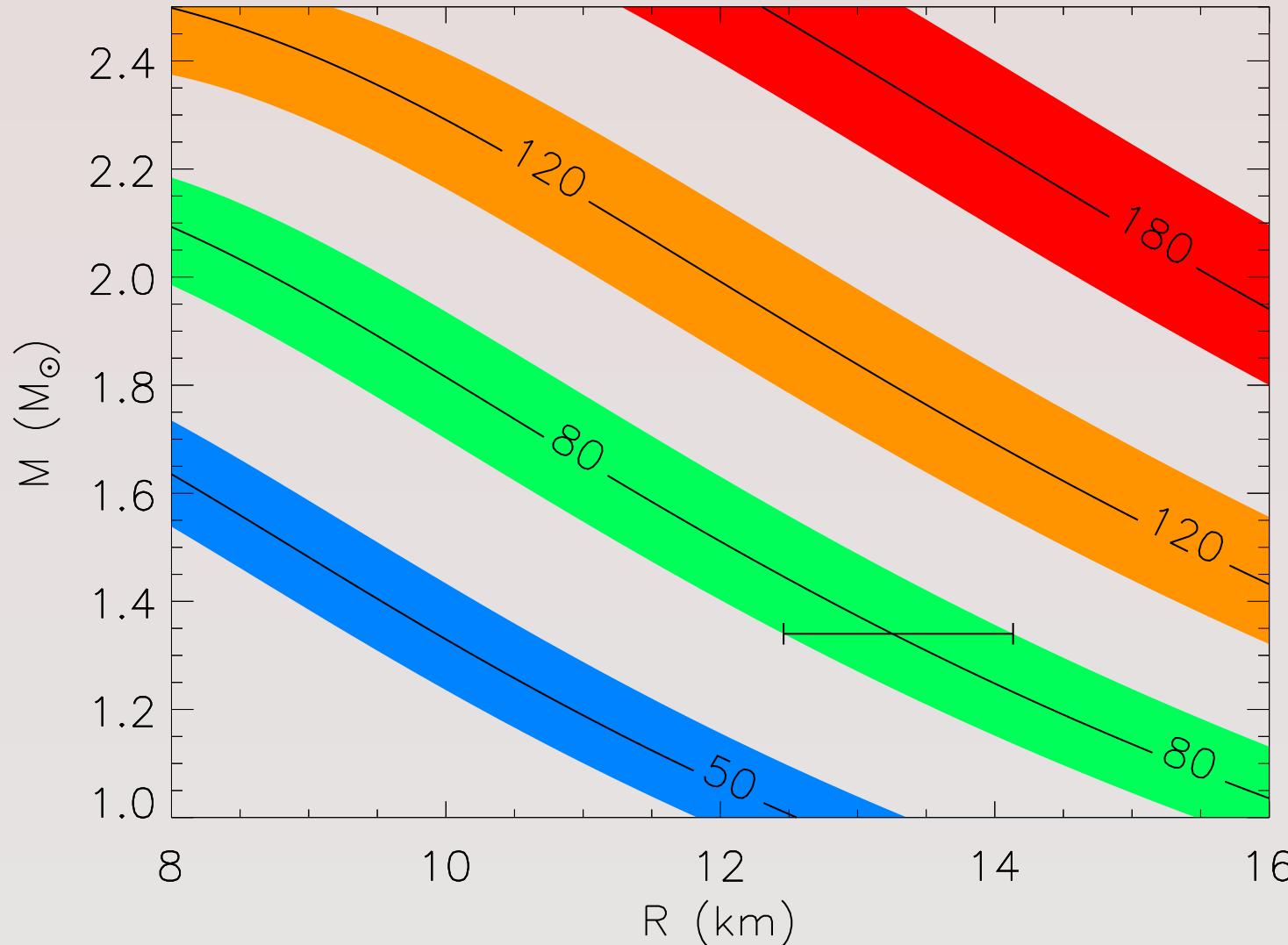
i : Orbital inclination angle θ_A : Angle between S_A and L.

I_A : Moment of Inertia of A

For PSR 0707-3039, $\delta t_A \simeq (0.17 \pm 0.16) I_{A,80} \mu\text{s}$;

Needs improved technology & is being pursued.

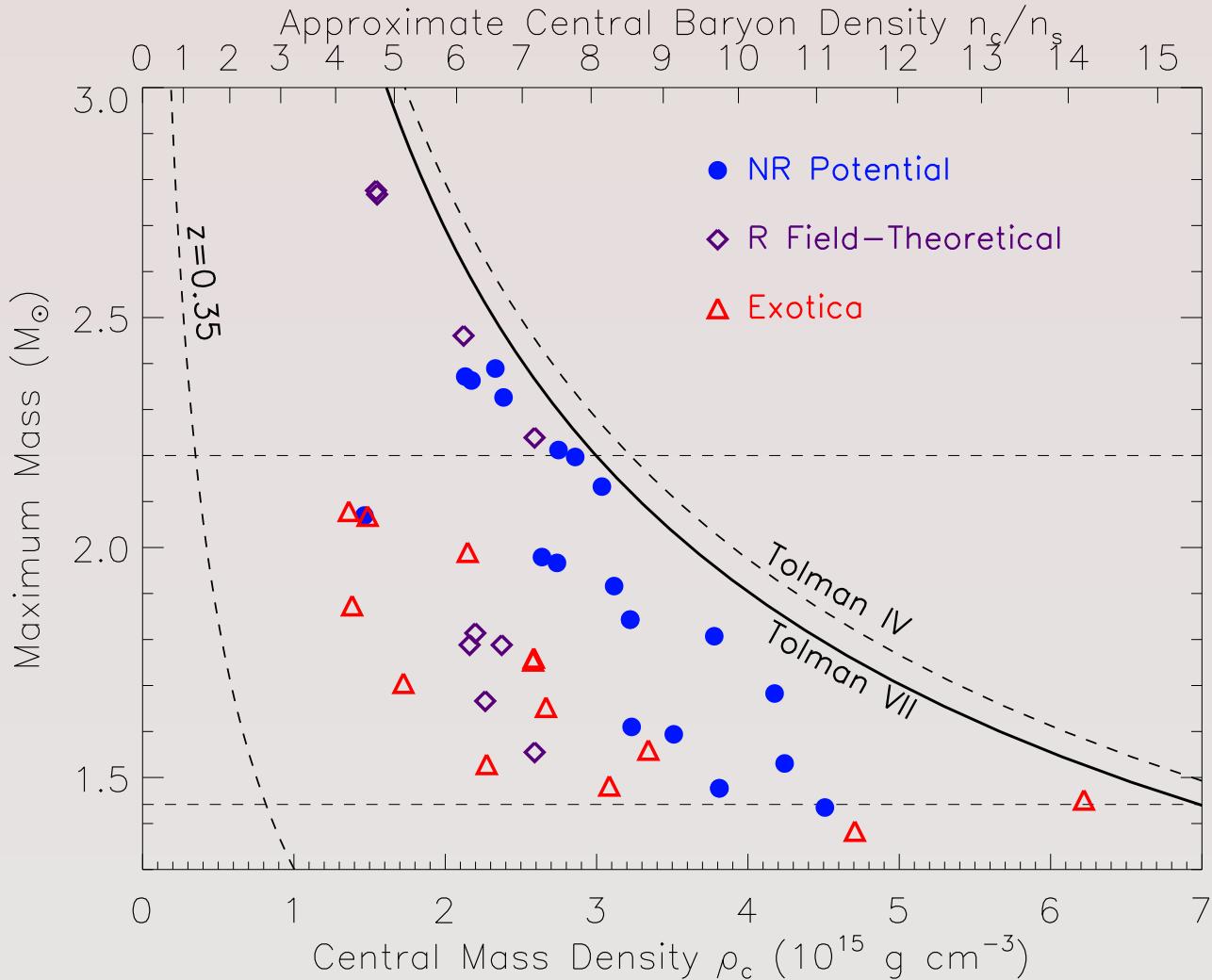
Limits on R from M & I measurements



- ▶ 10% error bands on I in $M_{\odot} \text{ km}^2$
- ▶ Horizontal error bar for $M = 1.34 M_{\odot}$ & $I = 80 \pm 8 M_{\odot} \text{ km}^2$

J. M. Lattimer & B. F. Schutz, *Astrophys. Jl.* **629** (2005)

Ultimate Energy Density of Cold Matter

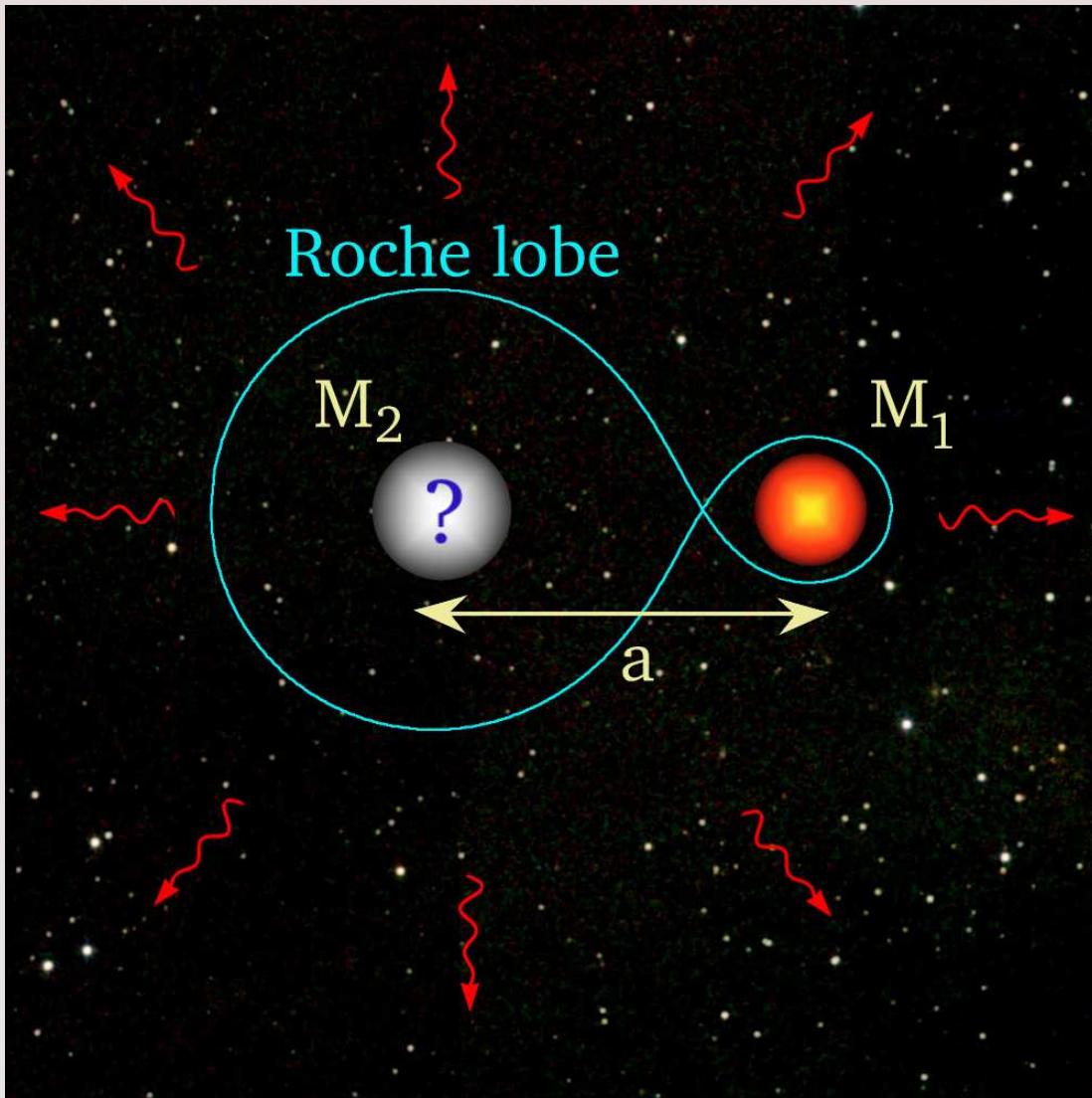


- ▶ Tolman VII:
 $\epsilon = \epsilon_c(1 - (r/R)^2)$
- ▶ $\epsilon_c \propto (M_\odot/M)^2$
- ▶ A measured red-shift provides a lower limit.
- ▶ Crucial to establish an upper limit to M_{max} .

Lattimer & Prakash, PRL, 94 (2005) 111101.

The Binary Merger Experience

The Ultimate Heavy-Ion Collision



- ▶ $M_1 \leq M_2$
- ▶ radial separation: $a(t)$
- ▶ M_1 - NS or SQM
- ▶ M_2 - BH, NS, ...
- ▶ GW emission \Rightarrow

$$\begin{aligned} L_{GW} &= \frac{1}{5} \frac{G}{c^5} \langle \ddot{\vec{x}}_{jk} \ddot{\vec{x}}_{jk} \rangle \\ &= \frac{32}{5} \frac{G^4}{c^5} \frac{M^3 \mu^2}{a^6} \end{aligned}$$

- orbit shrinks
- ▶ Mass transfer

Gravitational Wave Detection

- GW Strain : $h(t) = F_{\times}h_{\times}(t) + F_{+}h_{+}(t)$
 - $F_{\times,+}$: Constants of order unity
 - $h_{\times,+} \sim \frac{\delta L}{L_0} \sim \frac{1}{c^2} \frac{4G(E_{kin}^{ns}/c^2)}{r}$: Gravitational waveforms
 - L_0 : Unperturbed length of detector arm
 - δL : Relative change in length
 - E_{kin}^{ns} : Nonspherical part of the internal kinetic energy
 - ELF : $10^{-15} - 10^{-18}$ Hz VLF : $10^{-7} - 10^{-9}$ Hz*
 - LFB : 10^{-4} Hz - 1 Hz, HFB : 1 Hz - 10^4 Hz
- Astrophysical Sources Radiating GW's in the HFB

Supernovae	at 10 Mpc	$h \geq 10^{-25}$
Supernovae	Milky Way	$h \sim 10^{-18}$
$1.4M_{\odot}$ NS Binaries	at 10 Mpc	$h \sim 10^{-20}$
$10M_{\odot}$ BH Binaries	at 150 Mpc	$h \sim 10^{-20}$

Discovery of Double-Pulsar System

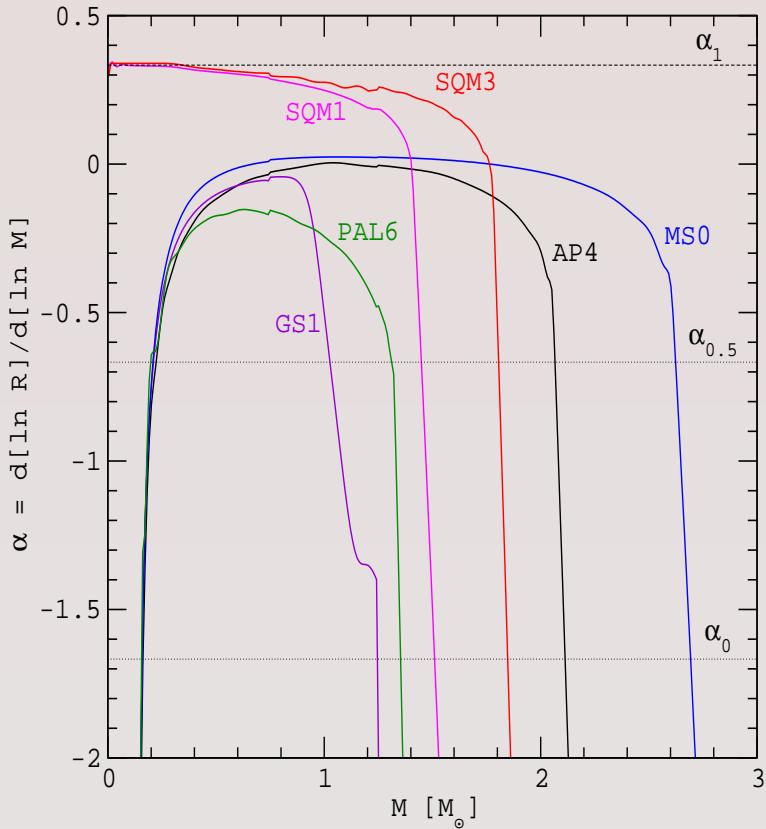
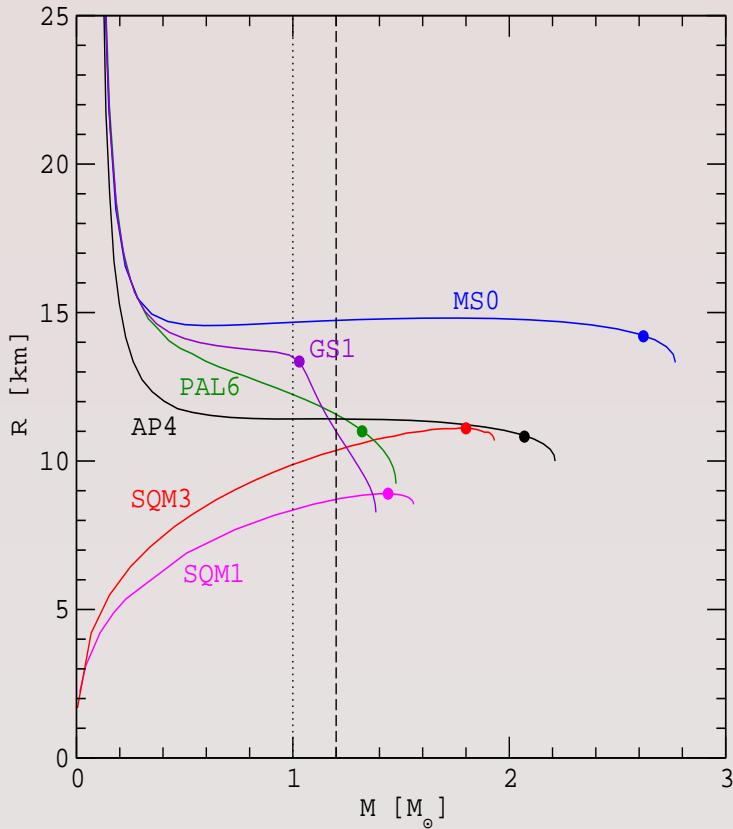
Pulsar	PSR J0737-3039A	PSR J0737-3039B
Pulse Period P (ms)	22.69937855615(6)	2773.4607474(4)
Period derivative \dot{P}	$1.74(5) \times 10^{-18}$	$0.88(13) \times 10^{-15}$
Orbital period P_b (day)	0.102251563(1)	—
Eccentricity e	0.087779(5)	—
Characteristic age (My)	210	50
Magnetic field B_s	6.3×10^9	1.6×10^{12}
Spin-down		
luminosity \dot{E} (erg/s)	5.8×10^{33}	1.6×10^{30}
Distance (kpc)	~ 0.6	—
Stellar mass	1.337(5)	1.250(5)

Merger expected in 85 Myr, a factor 3.5 shorter than PSR 1913+16

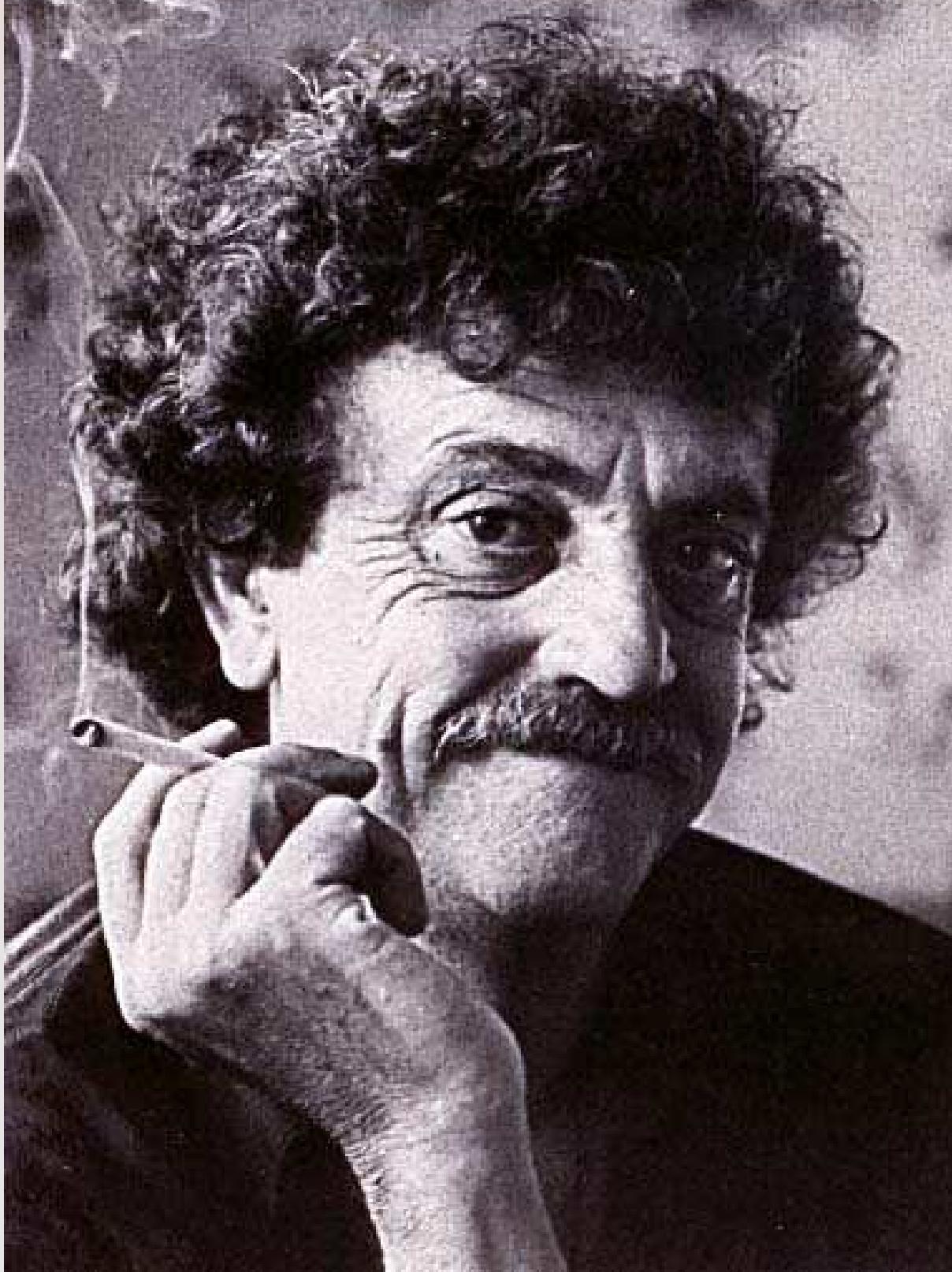
A.G Lyne et al., Science, 303, 1153 (2004)

Kalogera et al. (2004): Revisions w/ PSR J037-3039 imply 1 event per 1.5 yr for initial LIGO (for advanced LIGO, 20-1000 events per yr).

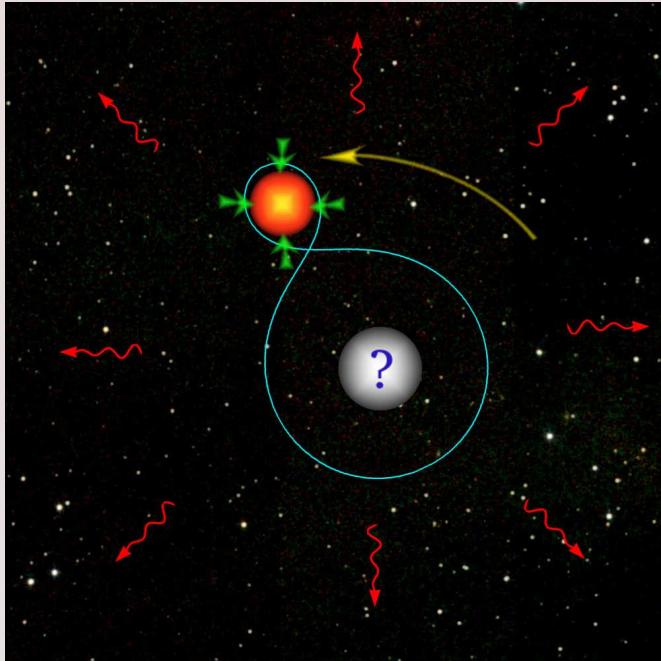
Equation of State: $\alpha(M)$



- ▶ EOS parameter : $\alpha = d \ln(R)/d \ln(M)$
- ▶ $\alpha_{NS} \leq 0$ (normal star)
- ▶ $\alpha_{SQM} \geq 0 (\approx 1/3)$ (self-bound star)



Roche Lobe Overflow



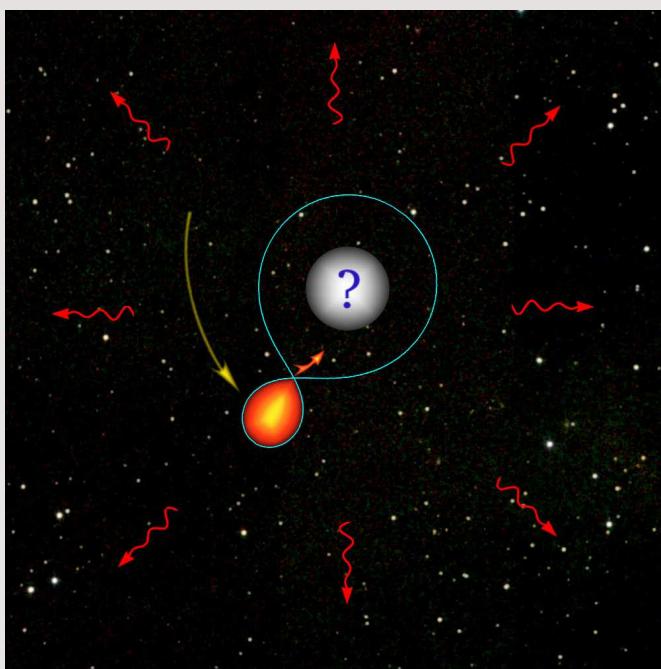
- Energy Loss

$$L_{GW} = \frac{1}{5} \langle \ddot{\mathcal{I}}_{jk} \ddot{\mathcal{I}}_{jk} \rangle = \frac{32}{5} a^4 \mu^2 \omega^6$$

- Angular Momentum Loss

$$(j_{GW})_i = \frac{2}{5} \epsilon_{ijk} \langle \ddot{\mathcal{I}}_{jm} \ddot{\mathcal{I}}_{km} \rangle = \frac{32}{5} a^4 \mu^2 \omega^5$$

- $a(t)$ and V_{Roche} shrink!

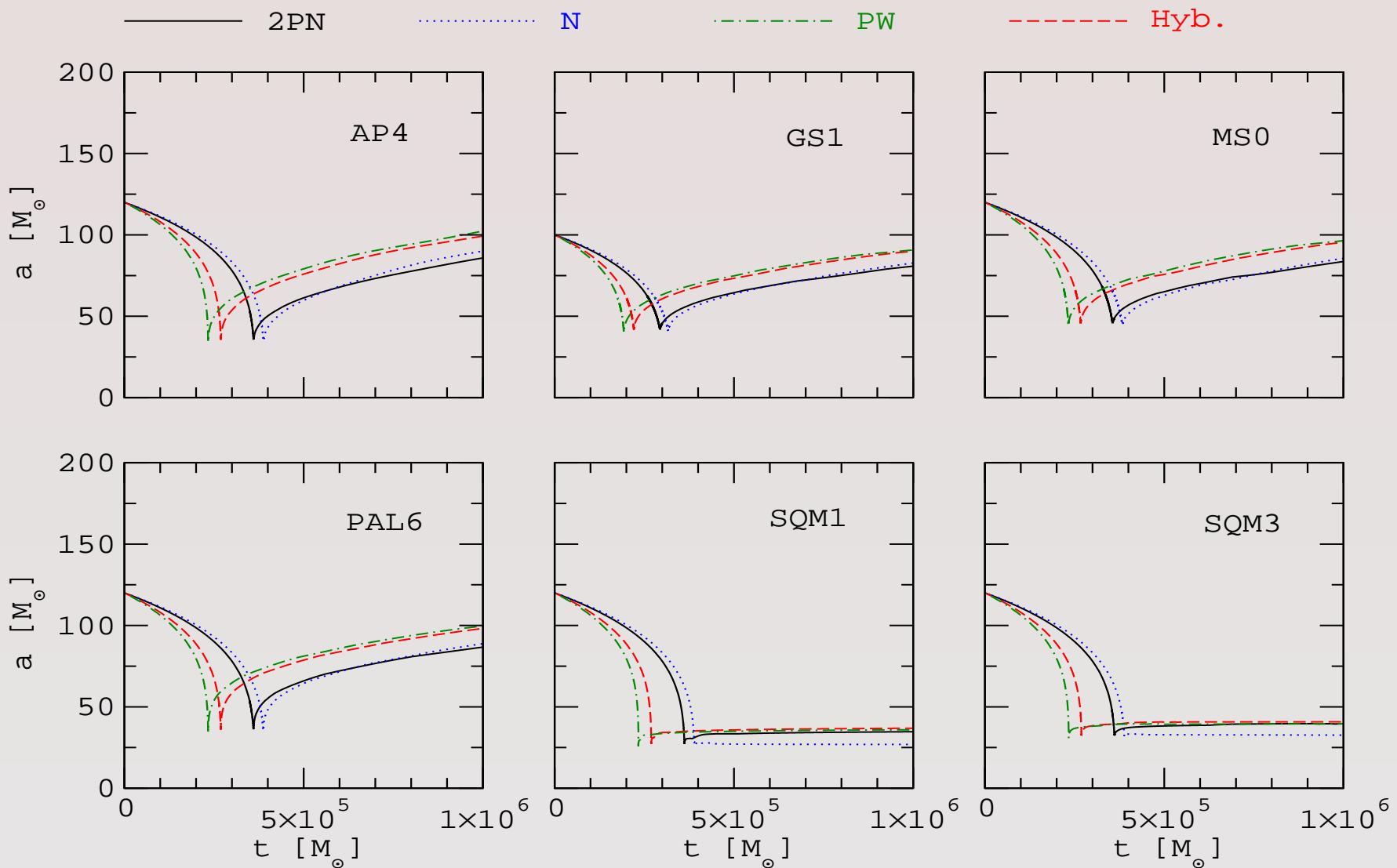


- $R_1 = r_{Roche}$

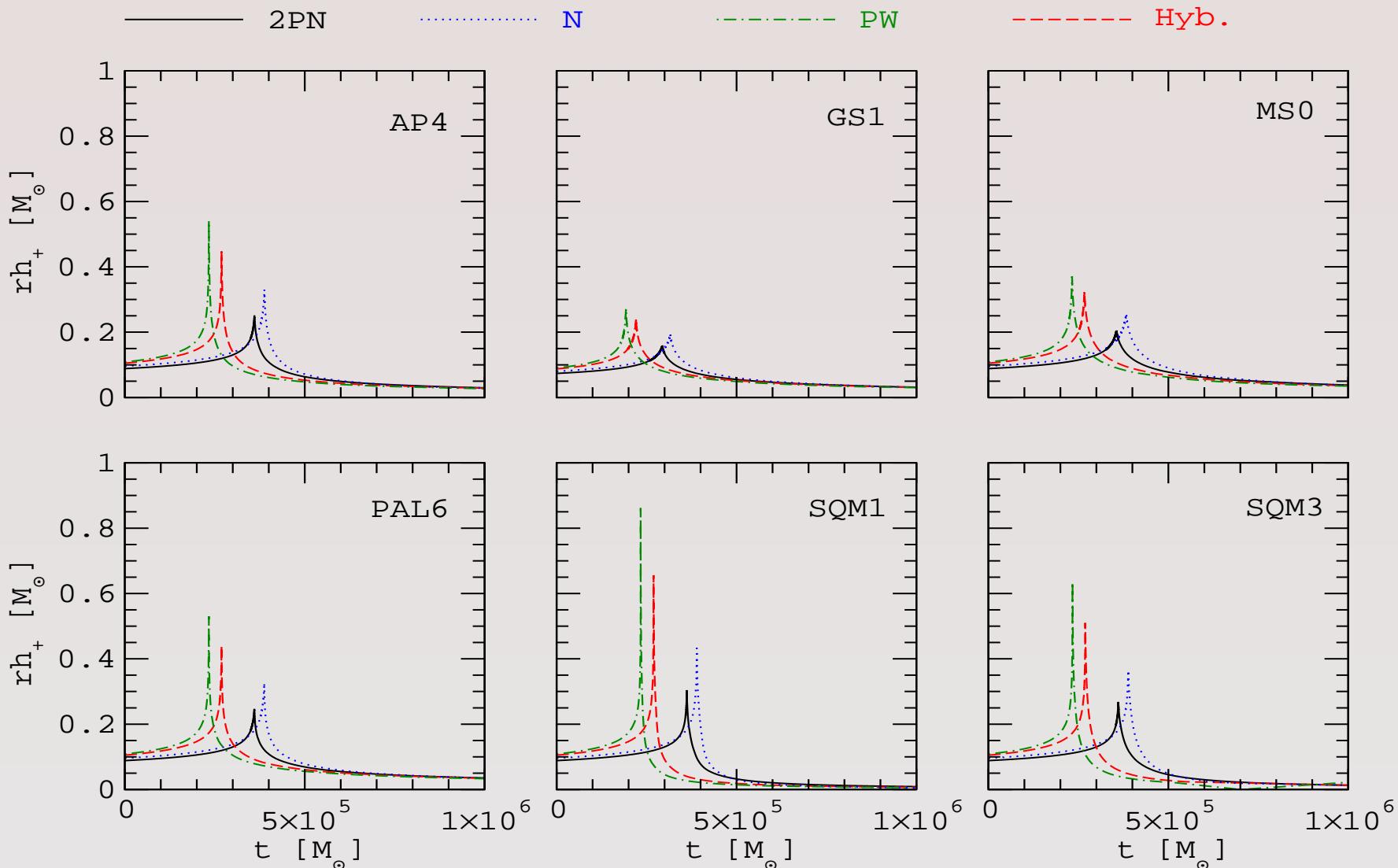
⇒ Mass transfer begins!

- To merge or not to merge?

Evolution: Orbit Separation a



Evolution: Distance \times Gravitational Amplitude rh_+



Major Results

- ▶ Incorporating GR into orbital dynamics leads to an evolution that is faster than the Newtonian evolution.
- ▶ Large differences exist between mergers of “normal” and “self-bound (SQM)” stars.
 - SQM stars penetrate to smaller orbital radii; stable mass transfer is more difficult than for normal stars.
 - For stable mass transfer, $q = M_1/M_2$ and $M = M_1 + M_2$ limits on SQM stars are more restrictive than for normal stars.
 - The SQM case has exponentially decaying signal and mass, while normal star evolution is slower.

Outlook

- ▶ Growing observations of neutron stars can delineate the equation of state of dense neutron-star matter that likely hosts quark matter.
- ▶ Given the promise of model calculations involving quark matter, lattice results at finite baryon density would be very welcome.
- ▶ A greater number of more accurate measurements, particularly of masses, radii, temperature, etc., are also necessary.





That's All Folks!